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The MathDBase project consists primarily of a comprehensive database of math concepts, with examples and hundreds of thousands of math problems solved in detail. The math courses covered are: Arithmetic, Algebra, Geometry, Trigonometry, Analytic Geometry, Pre-Calculus, Calculus, Probability & Statistics (Calculus-based), Linear Algebra and Ordinary Differential Equations. The MathDBase.com site is being developed and will include: structured self-study courses, worksheets, workbooks, videos and many other resources for math students and teachers.

The MathDBase project was conceived to provide as many resources as possible to anyone who aspires to a career that is in any way math-based.

> "No capable student who works hard and puts the time in should be 'weeded out' of their major of choice, due to a perceived 'lack of aptitude or talent.' Some students just need a bit more attention and take a little longer to blossom."

The project is extensive and help is needed to solve, check, edit and prepare 250,000-500,000 math problems for the database. Please help us by making a donation to PayPal.me/MathDBase. Thank you for your support.

MathDBase.com

All of the math you need to succeed!

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Multiplication Table

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2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Multiplying *any* number by 0 will give a product of 0. The entries on the main diagonal (in this color) are **perfect squares**, numbers that are the product of two equal integers ($3 \times 3 = 9$, $11 \times 11 = 121$ etc.).

Roman Numerals

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X
20	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**99**	**100**
XX	XXX	XL	L	LX	LXX	LXXX	XC	IC	C
200	**300**	**400**	**500**	**600**	**700**	**800**	**900**	**990**	**1,000**
CC	CCC	CD	D	DC	DCC	DCCC	CM	XM	M
50,000		500,000		1,000,000		1492		2014	
$\overline{\text{L}}$		$\overline{\text{D}}$		$\overline{\text{M}}$		MCDXCII		MMXIV	

Roman numerals were the symbols used for counting and mathematics in Ancient Rome. There is no counterpart for 0 in the Roman system. A pair of vertical bars around a Roman numeral mean 100 times that number ($|\text{V}| = \text{D} = 500$) and a horizontal bar over it means 1,000 times the number ($\overline{\text{V}} = 5{,}000$ and $|\overline{\text{V}}| = 500{,}000$).

Modern uses: II for 2nd and IV for 4th, etc. in titles and names; formal commemoration of dates on plaques and cornerstones of buildings.

Names of Large Numbers

Period name	Multiplier	Number of zeros
Thousand	10^3	3
Million	10^6	6
Billion	10^9	9
Trillion	10^{12}	12
Quadrillion	10^{15}	15
Quintillion	10^{18}	18
Sextillion	10^{21}	21
Septillion	10^{24}	24
Octillion	10^{27}	27
Nonillion	10^{30}	30
Decillion	10^{33}	33
Googol	10^{100}	100
Centillion	10^{303}	303
Googolplex	10^{Googol}	10^{100}

The value of a specific digit in a number is determined by its position, called its **place value**. The smallest value of a whole number is that of **Units** or **Ones**, the next is **Tens**, and the next position is **Hundreds**. Each position is ten times as large as the place to its immediate right. The three basic positions: Units, Tens and Hundreds, are called **orders**. A group of three orders is called a **period**. The first three periods are **Units**, **Thousands** and **Millions**. Each period contains a Units, Tens and Hundreds order, for example, there are millions, ten-millions and hundred-millions.

Metric System Prefixes

Prefix (Symbol)	Multiplier
Yotta (Y)	10^{24}
Zetta (Z)	10^{21}
Exa (E)	10^{18}
Peta (P)	10^{15}
Tera (T)	10^{12}
Giga (G)	10^{9}
Mega (M)	10^{6}
kilo (k)	10^{3}
hecto (h)	10^{2}
deka (da)	10
deci (d)	10^{-1}
centi (c)	10^{-2}
milli (m)	10^{-3}
micro (μ)	10^{-6}
nano (n)	10^{-9}
pico (p)	10^{-12}
femto (f)	10^{-15}
atto (a)	10^{-18}
zepto (z)	10^{-21}
yocto (y)	10^{-24}

To apply a prefix, multiply the basic unit by the multiplier. Example: 3 kilometers = 3×10^3 meters = $3 \times 1{,}000$ meters = 3,000 meters. 2.5 Terabytes = 2.5×10^{12} bytes = 2,500,000,000,000 bytes.

Measurements of Time

Time Measurements and Equivalents	
60 seconds (sec)	1 minute (min)
60 minutes	1 hour (hr)
24 hours	1 day (da)
7 days	1 week (wk)
28-31 days	1 month (mo)
365 days	1 year (yr)
52 weeks (approximate)	
12 months	
366 days	1 leap year
10 years	1 decade
20 years	1 score
100 years	1 century (C)
1,000 years	1 millennium (M)

Number of days in each calendar month	
28	February
29	February (leap year)
30	April, June, September, November
31	January, March, May, July, August, October, December

English and Metric Measures of Weight

English System of Weights	
437.5 grains (gr)	1 ounce (oz)
7,000 grains	1 pound (lb)
16 ounces	
2,000 pounds	1 short ton (ST)
2,240 pounds	1 long (or gross) ton (LT)
1.12 short tons	1 long ton

Also called **Avoirdupois** weight, this system is mainly used to weigh heavy industrial, wholesale commercial and farming products, such as coal, steel, chemicals, livestock and grain. The **stone** is a unit primarily used in Great Britain: 1 stone = 14 pounds.

Metric System of Weights		
10 milligrams (mg)	1 centigram (cg)	0.01 grams (g)
10 centigrams	1 decigram (dg)	0.1 grams
10 decigrams	1 gram	
10 grams	1 decagram (Dg)	10 grams
10 decagrams	1 hectogram (hg)	100 grams
10 hectograms	1 kilogram (kg)	1,000 grams
10 kilograms	1 myriagram (Mg)	10,000 grams
10 myriagrams	1 quintal (Q)	100,000 grams
10 quintals	1 metric tonne (T)	1,000,000 grams

Used primarily in scientific work (**SI** units - International System) and to a much lesser degree in packaging food stuffs.

English-Metric Weight and Temperature Conversions

English-Metric Conversions		Metric-English Conversions	
1 gr	0.064799 g	1 g	15.432 gr
1 oz	28.35 g		0.035274 oz
1 lb	0.45359 kg	1 kg	2.2046 lb
1 ST	907.18 kg	1 T	0.98421 LT
	0.90718 T		
1 LT	1,016 kg		1.1023 ST
	1.016 T		

Measures of Temperature

Fahrenheit (F) to Celsius (C) Conversions:

$$C = \frac{5}{9}\left(F - 32^\circ\right)$$

Celsius (C) to Fahrenheit (F) Conversions:

$$F = \frac{9}{5}C + 32^\circ$$

Some special temperatures	Fahrenheit	Celsius
Average room temperature	68°	20°
Average "normal" body temperature	98.6°	37°
Freezing point of water	32°	0°
Boiling point of water (sea lev.)	212°	~100°

English and Metric Measures of Length and Distance

Units of Linear Measurement - English System	
12 inches (in)	1 foot (ft)
3 feet	1 yard (yd)
5 ½ yards	1 rod (rd)
16 ½ feet	
320 rods	1 mile (mi)
1,760 yards	
5,280 feet	

Units of Linear Measurement - Metric System		
1,000 millimeters (mm)	1 meter (m)	1.0 meter
10 millimeters	1 centimeter (cm)	0.01 meter
10 centimeters	1 decimeter (dm)	0.1 meter
100 centimeters	1 meter	1.0 meter
10 decimeters		
10 meters	1 decameter (Dm/ dam)	10 meters
10 decameters	1 hectometer (hm)	100 meters
10 hectometers	1 kilometer (km)	1,000 meters

Measures of Length and Distance - Conversions

English-Metric Conversions		Metric-English Conversions	
1 in	2.54 cm	1 mm	0.03937 in
	25.4 mm	1 cm	0.3937 in
39.37 in	1 m		0.0328 ft
1 ft	0.3048 m	1 m	39.37 in
	30.48 cm		3.2808 ft
1 yd	0.9144 m		1.0936 yd
1 mi	1.6093 km	1 km	3,280.8 ft
	1,609.3 m		1,093.6 yd
0.62137 mi	1 km		0.62137 mi

Conversion Multipliers (approximate)					
English to Metric			**Metric to English**		
Multiply	**by**	**to get...**	**Multiply**	**by**	**to get...**
in	25.4	mm	mm	0.0394	in
	2.54	cm	cm	0.394	
ft	0.304	m	m	3.28	ft
yd	0.91			1.09	yd
mi	1.62137	km	km	0.62137	mi

First 300 Prime Numbers

A **prime** number is a natural number (see p. 17) larger than 1, whose only divisors are 1 and itself. A **composite** number is a natural number that is not prime. 2 is the *only* even prime number.

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47
53	59	61	67	71	73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173	179	181	191	193	197
199	211	223	227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359	367	373	379
383	389	397	401	409	419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541	547	557	563	569	571
577	587	593	599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743	751	757	761
769	773	787	797	809	811	821	823	827	829	839	853	857	859	863
877	881	883	887	907	911	919	929	937	941	947	953	967	971	977
983	991	997	1009	1013	1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
1087	1091	1093	1097	1103	1109	1117	1123	1129	1151	1153	1163	1171	1181	1187
1193	1201	1213	1217	1223	1229	1231	1237	1249	1259	1277	1279	1283	1289	1291
1297	1301	1303	1307	1319	1321	1327	1361	1367	1373	1381	1399	1409	1423	1427
1429	1433	1439	1447	1451	1453	1459	1471	1481	1483	1487	1489	1493	1499	1511
1523	1531	1543	1549	1553	1559	1567	1571	1579	1583	1597	1601	1607	1609	1613
1619	1621	1627	1637	1657	1663	1667	1669	1693	1697	1699	1709	1721	1723	1733
1741	1747	1753	1759	1777	1783	1787	1789	1801	1811	1823	1831	1847	1861	1867
1871	1873	1877	1879	1889	1901	1907	1913	1931	1933	1949	1951	1973	1979	1987

Prime Factorizations of the first 132 Composite Numbers

$4 = 2^2$	$6 = 2 \cdot 3$	$8 = 2^3$	$9 = 3^2$	$10 = 2 \cdot 5$	$12 = 2^2 \cdot 3$
$14 = 2 \cdot 7$	$15 = 3 \cdot 5$	$16 = 2^4$	$18 = 2 \cdot 3^2$	$20 = 2^2 \cdot 5$	$21 = 3 \cdot 7$
$22 = 2 \cdot 11$	$24 = 2^3 \cdot 3$	$25 = 5^2$	$26 = 2 \cdot 13$	$27 = 3^3$	$28 = 2^2 \cdot 7$
$30 = 2 \cdot 3 \cdot 5$	$32 = 2^5$	$33 = 3 \cdot 11$	$34 = 2 \cdot 17$	$35 = 5 \cdot 7$	$36 = 2^2 \cdot 3^2$
$38 = 2 \cdot 19$	$39 = 3 \cdot 13$	$40 = 2^3 \cdot 5$	$42 = 2 \cdot 3 \cdot 7$	$44 = 2^2 \cdot 11$	$45 = 3^2 \cdot 5$
$46 = 2 \cdot 23$	$48 = 2^4 \cdot 3$	$49 = 7^2$	$50 = 2 \cdot 5^2$	$51 = 3 \cdot 17$	$52 = 2^2 \cdot 13$
$54 = 2 \cdot 3^3$	$55 = 5 \cdot 11$	$56 = 2^3 \cdot 7$	$57 = 3 \cdot 19$	$58 = 2 \cdot 29$	$60 = 2^2 \cdot 3 \cdot 5$
$62 = 2 \cdot 31$	$63 = 3^2 \cdot 7$	$64 = 2^6$	$65 = 5 \cdot 13$	$66 = 2 \cdot 3 \cdot 11$	$68 = 2^2 \cdot 17$
$69 = 3 \cdot 23$	$70 = 2 \cdot 5 \cdot 7$	$72 = 2^3 \cdot 3^2$	$74 = 2 \cdot 37$	$75 = 3 \cdot 5^2$	$76 = 2^2 \cdot 19$
$77 = 7 \cdot 11$	$78 = 2 \cdot 3 \cdot 13$	$80 = 2^4 \cdot 5$	$81 = 3^4$	$82 = 2 \cdot 41$	$84 = 2^2 \cdot 3 \cdot 7$
$85 = 5 \cdot 17$	$86 = 2 \cdot 43$	$87 = 3 \cdot 29$	$88 = 2^3 \cdot 11$	$90 = 2 \cdot 3^2 \cdot 5$	$91 = 7 \cdot 13$
$92 = 2^2 \cdot 23$	$93 = 3 \cdot 31$	$94 = 2 \cdot 47$	$95 = 5 \cdot 19$	$96 = 2^5 \cdot 3$	$98 = 2 \cdot 7^2$
$99 = 3^2 \cdot 11$	$100 = 2^2 \cdot 5^2$	$102 = 2 \cdot 3 \cdot 17$	$104 = 2^3 \cdot 13$	$105 = 3 \cdot 5 \cdot 7$	$106 = 2 \cdot 53$
$108 = 2^2 \cdot 3^3$	$110 = 2 \cdot 5 \cdot 11$	$111 = 3 \cdot 37$	$112 = 2^4 \cdot 7$	$114 = 2 \cdot 3 \cdot 19$	$115 = 5 \cdot 23$
$116 = 2^2 \cdot 29$	$117 = 3^2 \cdot 13$	$118 = 2 \cdot 59$	$119 = 7 \cdot 17$	$120 = 2^3 \cdot 3 \cdot 5$	$121 = 11^2$
$122 = 2 \cdot 61$	$123 = 3 \cdot 41$	$124 = 2^2 \cdot 31$	$125 = 5^3$	$126 = 2 \cdot 3^2 \cdot 7$	$128 = 2^7$
$129 = 3 \cdot 43$	$130 = 2 \cdot 5 \cdot 13$	$132 = 2^2 \cdot 3 \cdot 11$	$133 = 7 \cdot 19$	$134 = 2 \cdot 67$	$135 = 3^3 \cdot 5$
$136 = 2^3 \cdot 17$	$138 = 2 \cdot 3 \cdot 23$	$140 = 2^2 \cdot 5 \cdot 7$	$141 = 3 \cdot 47$	$142 = 2 \cdot 71$	$143 = 11 \cdot 13$
$144 = 2^4 \cdot 3^2$	$145 = 5 \cdot 29$	$146 = 2 \cdot 73$	$147 = 3 \cdot 7^2$	$148 = 2^2 \cdot 37$	$150 = 2 \cdot 3 \cdot 5^2$
$152 = 2^3 \cdot 19$	$154 = 2 \cdot 7 \cdot 11$	$155 = 5 \cdot 31$	$156 = 2^2 \cdot 3 \cdot 13$	$158 = 2 \cdot 79$	$159 = 3 \cdot 53$
$160 = 2^5 \cdot 5$	$161 = 7 \cdot 23$	$162 = 2 \cdot 3^4$	$164 = 2^2 \cdot 41$	$165 = 3 \cdot 5 \cdot 11$	$166 = 2 \cdot 83$
$168 = 2^3 \cdot 3 \cdot 7$	$170 = 2 \cdot 5 \cdot 17$	$171 = 3^2 \cdot 19$	$172 = 2^2 \cdot 43$	$174 = 2 \cdot 3 \cdot 29$	$175 = 5^2 \cdot 7$

Percentage, Percentage Increase and Decrease, Interest

Basic Percentage Formulas

Percentage (P), Rate (R), Base (B), Amount (A), Difference (D)

Percentage is equal to Rate times Base: $P = RB$

Amount is equal to Base plus Percentage: $A = B + P$

Difference is equal to Base minus Percentage: $D = B - P$

Base is equal to the quotient of the sum of the Rate as a decimal and 1: $B = A/(R + 1)$

Rate is equal to the quotient of the difference between the Amount and the Base, and the Base: $R = (A - B)/B$

Percentage Increases and Decreases

Base is equal to the Difference and the difference between 1 and the Rate as a decimal: $B = D/(1 - R)$

Rate is equal to the quotient of the difference between the Base and Difference, and the Base: $R = (B - D)/B$

Simple Interest

Interest (I), Principal (P), Interest Rate (r), Time (t)

Interest is equal to Principal times Interest Rate times Time: $I = Prt$

Sum is the total of the Principal and Interest: $S = P + I = P + Prt = P(1 + rt)$

Compound Interest

A Sum is accumulated based on a Principal and interest rate over time, in years: $S = P(1 + r)^t$

Algebra - Sets of Numbers

Symbol	Name	Description	Set Notation
$\mathbb{N}$	Natural Numbers	Counting numbers	$\{1,2,3,...\}$
$\mathbb{W}$	Whole Numbers	Counting numbers and 0	$\{0,1,2,...\}$
$\mathbb{Z}$	Integers	Positive and negative Whole Numbers	$\{...,-1,0,1,...\}$
$\mathbb{Q}$	Rational Numbers	Includes all fractions, terminating and repeating decimals	$\{p/q \mid p, q \in \mathbb{Z}$ $q \neq 0\}$
$\mathbb{I}$	Irrational Numbers	Numbers that cannot be written as fractions (e.g. π & e)	$\{x \mid x \neq p/q\}$
$\mathbb{R}$	Real Numbers	Includes *all* of the numbers above	$\{x \mid x \in \mathbb{Q}$ or $x \in \mathbb{I}\}$
$\mathbb{C}$	Complex Numbers	Contains the Real Numbers as a proper subset	$\{a + bi \mid$ $a, b \in \mathbb{R}$ $i^2 = -1\}$

For all real numbers *a*, *b* and *c*:

Laws of Addition

1 Commutative Law of Addition $a+b=b+a$

2 Associative Law of Addition $a+(b+c)=(a+b)+c$

3 Additive Identity $a+0=a=0+a$

4 Additive Inverse $a+(-a)=0$

Laws of Multiplication

1 Commutative Law of Multiplication $ab=ba$

2 Associative Law of Multiplication $a(bc)=(ab)c$

3 Multiplicative Identity $a\cdot 1=a=1\cdot a$

4 Multiplicative Inverse $a\cdot\frac{1}{a}=1=\frac{1}{a}\cdot a, a\neq 0$

The **absolute value** $|x|$ of a number is x, if x is positive, $-x$, if x is negative and 0 if x is 0.

Addition and Subtraction of Fractions

1 $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}; \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}, b, d \neq 0$

2 $a+\frac{b}{c}=\frac{ac+b}{c}, c \neq 0; a-\frac{b}{c}=\frac{ac-b}{c}, c \neq 0$

3 $\frac{a}{b}+c=\frac{a+bc}{b}, b \neq 0; \frac{a}{b}-c=\frac{a-bc}{b}, b \neq 0$

Multiplication and Division of Fractions

1 $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}=\frac{ac}{bd}, b, d \neq 0$

2 $\frac{a}{b}\div\frac{c}{d}=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}, b, c, d \neq 0$

The Order of Operations

Perform operations inside of grouping symbols first, if there are no grouping symbols then: **1** simplify all radicals, exponentials and absolute values; **2** multiply and divide, from left to right; **3** add and subtract, from left to right.

Algebra - Rules of Exponents

1 Zero Exponent Rule $a^0 = 1, a \neq 0$

2 Product Rule $a^m \cdot a^n = a^{m+n}$

3 Power Rule $\left(a^m\right)^n = a^{mn}$

4 Power of a Product Rule $\left(a \cdot b\right)^n = a^n \cdot b^n$

5 Power of a Quotient Rule $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

6 Quotient Rule $\frac{a^m}{a^n} = a^{m-n}, m > n, a \neq 0$

7 Negative Exponent Rule a $a^{-n} = \frac{1}{a^n}, a \neq 0$

8 Negative Exponent Rule b $\frac{1}{a^{-n}} = a^n, a \neq 0$

9 Negative Exponent Rule c $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}, a, b \neq 0$

10 Radicals-Absolute Value $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$; if n is even and $m = n$, then $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^n = |a|$ (see p. 18).

Algebra - Binomial Expansions, Pascal's Triangle

Some Basic Binomial Expansions

1 $(a+b)^0 = 1$ **2** $(a+b)^1 = 1a + 1b$

3 $(a-b)^1 = 1a - 1b$

4 $(a+b)^2 = 1a^2 + 2ab + 1b^2$

5 $(a-b)^2 = 1a^2 - 2ab + 1b^2$

6 $(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

7 $(a-b)^3 = 1a^3 - 3a^2b + 3ab^2 - 1b^3$

$\left(a+b\right)^n = a^n + na^{n-1}b + \cdots + {}_nC_r a^{n-r}b^r + \cdots + nab^{n-1} + b^n$, where the coefficient of $a^{n-r}b^r$ is given by ${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ (see p. 22).

If the expansion has $\left(a-b\right)^n$ form, rewrite it as $\left[a+\left(-b\right)\right]^n$ first.

Pascal's Triangle

Among many other applications, each row of the triangle gives the coefficients of a binomial expansion. Other than the edge coefficients (1s), each entry is the sum of the two entries to its immediate right and left in the row above it. The number of entries in a row is always one more than the power of the expansion.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Perfect Square Trinomial $a^2 + 2ab + b^2 = (a + b)^2$

Perfect Square Trinomial $a^2 - 2ab + b^2 = (a - b)^2$

Difference of Squares $a^2 - b^2 = (a + b)(a - b)$

Sum of Squares $a^2 + b^2 = (a + bi)(a - bi)$, where $i^2 = -1$

Difference of Cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Sum of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Factorial Formulas

1 $0! = 1$ **2** $n! = n(n-1)(n-2)\cdots 2\cdot 1$, where $n \in \mathbb{N}$

Binomial Coefficients

$$\binom{n}{r} = \frac{n(n-1)(n-2)\cdots[n-(r-1)]}{1\cdot 2\cdot 3\cdots r} = \frac{n!}{r!(n-r)!},$$

where $n \in \mathbb{N}, 0 < r \le n$

Properties of the Binomial Coefficients

$$\binom{n}{0} = 1 = \binom{n}{n}, \binom{n}{1} = n, \binom{n}{r} = \binom{n}{n-r}, \binom{n}{r} + \binom{n}{r+1}$$

$$= \binom{n+1}{r+1}, \text{ where } r, n \in \mathbb{N}$$

$ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$, is a quadratic equation in **general form**. A quadratic equation will have at most two real solutions, which can be found, if factorable, using a product of linear factors $(x - a)(x - b) = 0$ with the solutions $x_1 = a$ and $x_2 = b$, or generally, using the **quadratic formula**:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The expression $b^2 - 4ac$ is called the **discriminant**. If a, b and c are rational numbers, the roots of the quadratic equation are:

a real and equal, if and only if $b^2 - 4ac = 0$

b real and unequal if and only if $b^2 - 4ac > 0$

c imaginary and unequal if and only if $b^2 - 4ac < 0$

d rational if and only if $b^2 - 4ac$ is the square of a rational number.

If x_1 and x_2 are the roots, then $x_1 + x_2 = -\frac{b}{a}$ and $x_1 \cdot x_2 = \frac{c}{a}$.

$x^2 + px + q = 0$, where p and q are real numbers, is a quadratic equation in **normal form**, its solutions can be found using the formula:

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left[\left(\frac{p}{2}\right)^2 - q\right]}.$$

Algebra - Logarithmic and Exponential Functions

If a, b, c, B, C and $D > 0$:

Logarithm to any base of 1 $\log_a(1) = 0$

Logarithm to any base of the base $\log_a(a) = 1$

Logarithm to any base of 0 $\log_a(0) = -\infty$

Logarithm of a Product $\log_a(C \cdot D) = \log_a(C) + \log_a(D)$

Logarithm of a Quotient $\log_a(C/D) = \log_a(C) - \log_a(D)$

An argument to an exponent 1 $\log_a(B^n) = n\log_a(B)$

An argument to an exponent 2 $\log_a(B^{n/m}) = (n/m)\log_a(B)$

Change of Base $\log_b(a) = \log_c(a)/\log_c(b)$

Reciprocal Relationship $\log_a(b) = 1/\log_b(a)$

Conversion Multiplier $\log_b(a) = k\log_c(a),\ k \in \mathbb{R}$

Product of Logarithms $\log_a(b) \cdot \log_b(c) = \log_a(c)$

If $f(x) = k^x$, where $k, x > 0$ and $g(x) = \log_k(x)$, then $f(x)$ and $g(x)$ are inverse functions and:

1 $f(g(x)) = k\text{^}[\log_k(x)] = x$, where $k\text{^}u = k^u$

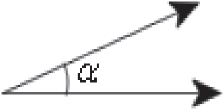
2 $g(f(x)) = \log_k(k^x) = x\log_k(k) = x \cdot 1 = x$

If $k > 0$, then $k^x = e^{x\ln(k)}$, where $\ln(k) = \log_e(k)$ - **natural logarithm**, with base $e \approx 2.718281828...$

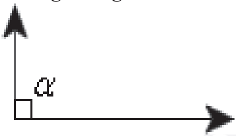
Geometry - Basic Angles

Zero Angle $\alpha = 0°$

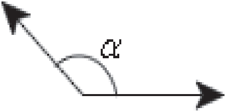
Acute Angle $\alpha < 90°$



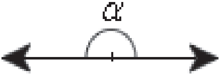
Right Angle $\alpha = 90°$



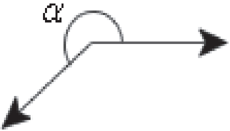
Obtuse Angle $90° < \alpha < 180°$



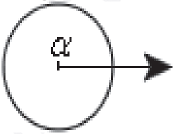
Straight Angle $\alpha = 180°$



Reflex Angle $180° < \alpha < 360°$



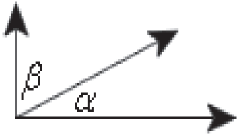
Full Angle $\alpha = 360°$



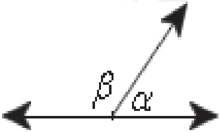
Angles over 360° $\alpha > 360°$

Geometry - Angle Pairs, Parallel Lines, Transversals

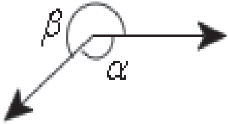
Complementary Angles $\alpha + \beta = 90°$



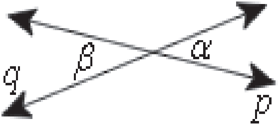
Supplementary Angles $\alpha + \beta = 180°$



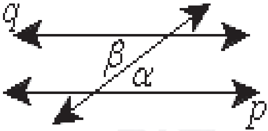
Explementary Angles $\alpha + \beta = 360°$



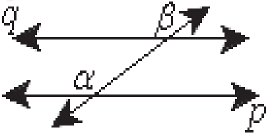
Vertical Angles $\alpha = \beta$



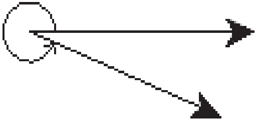
Alternate Interior Angles $p \parallel q$, $\alpha = \beta$



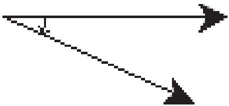
Corresponding Ext. Angles $p \parallel q$, $\alpha = \beta$



Positive Angle



Negative Angle

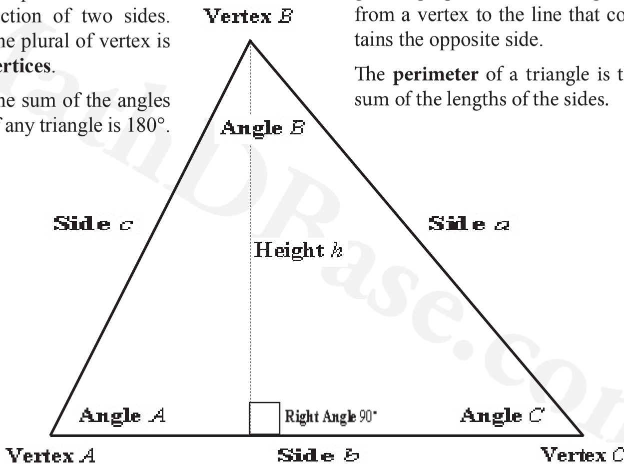


A **vertex** of a triangle is a point at the intersection of two sides. The plural of vertex is **vertices**.

The sum of the angles of any triangle is 180°.

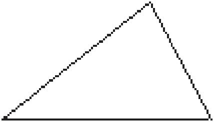
An **altitude** (or **height**) of a triangle is a perpendicular line segment from a vertex to the line that contains the opposite side.

The **perimeter** of a triangle is the sum of the lengths of the sides.

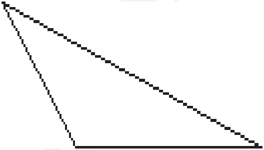


Geometry - Types of Triangles

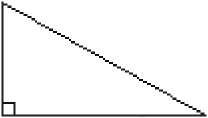
Acute triangles have three acute interior angles.



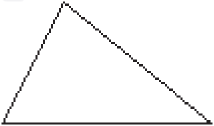
Obtuse triangles have one obtuse angle and two acute angles.



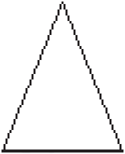
Right triangles have one right angle and two acute angles.



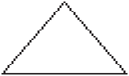
Scalene triangles have three sides of different lengths and three interior angles with different measures.



Isosceles triangles have at least two sides of equal length and two interior angles with equal measure.



Equilateral triangles have three sides of equal length. Each interior angle measures 60°.



Geometry - Triangle Formulas

General Triangles

The area A of a triangle is equal to half of the product of a base and the height: $A = \frac{1}{2}bh.$

Also, $A = \frac{1}{2}bc\sin(A) = \frac{1}{2}ac\sin(B) = \frac{1}{2}ab\sin(C)$, where a, b and c represent the lengths of the sides of the triangle (see p. 27) and A, B and C represent the measures of the angles. See p. 34 for the definitions of sine (sin) and cosine (cos).

Heron's Formula: If a, b and c represent the lengths of the sides of a triangle and P its perimeter, then the area is

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $P = a + b + c$ and $s = \frac{P}{2}$.

If a, b and c represent the lengths of the sides of a triangle (see p. 27) and A, B and C the measures of the angles:

The Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

The Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

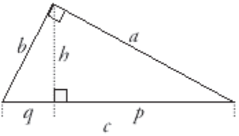
Geometry - Right, Isosceles and Equilateral Triangles

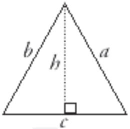
Right Triangles

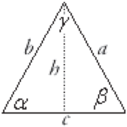
Pythagorean Theorem: The square of the length *c* of the hypotenuse is equal to the sum of the squares of the lengths *a* and *b*, of the other two sides: $c^2 = a^2 + b^2$. **Area**: $A = \frac{1}{2}ab$

Euclidean Formulas: **1** $h^2 = pq$ **2** $a^2 = cp$ **3** $b^2 = cq$

Right **Isosceles** **Equilateral**







Isosceles Triangles

In an isosceles triangle, *at least* two sides have equal length. $a = b$ above. The angles opposite those sides have equal measure.

The altitude *h* or height is $h = \sqrt{a^2 - \left(\frac{c}{2}\right)^2}$.

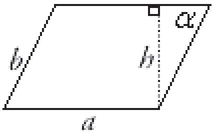
Equilateral Triangles

Equilateral triangles have three sides of equal length and three acute angles of equal measure, so equilateral triangles are isosceles triangles. Since the sum of the measures of the internal angles of a triangle ($\alpha + \beta + \gamma$) is 180°, each acute angle measures 60°. $a = b = c$,

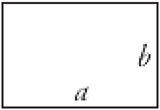
so the altitude is $h = \frac{\sqrt{3}}{2}a$ and the area is $A = \frac{\sqrt{3}}{4}a^2$

Geometry - Types of Quadrilaterals

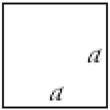
Parallelograms have opposite sides that are parallel.



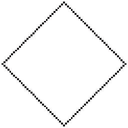
Rectangles are parallelograms with four right angles and opposite sides of equal length.



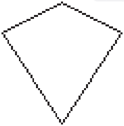
Squares are rectangles with four sides of equal length.



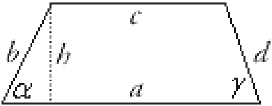
Rhombuses are parallelograms that have four sides of equal length.



Kites are quadrilaterals with two pairs of equal-length adjacent sides.



Trapezoids are quadrilaterals with at least one pair of parallel sides.



Geometry - Quadrilateral Formulas and Properties

Parallelograms

Perimeter: $P = 2a + 2b = 2a + \frac{2h}{\sin(\alpha)}$

Area: $A = ah = b\sin(\alpha)$

The diagonals of a parallelogram bisect each other. Rectangles, squares and rhombuses are special types of parallelograms.

Rectangles

Perimeter: $P = 2a + 2b = 2(a+b)$

Area: $A = ab$

The diagonals of a rectangle bisect each other and have equal length. The sum of the internal angles of any quadrilateral is 360°.

Squares

Perimeter: $P = 2a + 2b = 2(a+b)$

Area: $A = ab$

A square is a rectangle with two pairs of adjacent sides of equal length. A square is a rhombus with all angles of equal size (90°).

Trapezoids

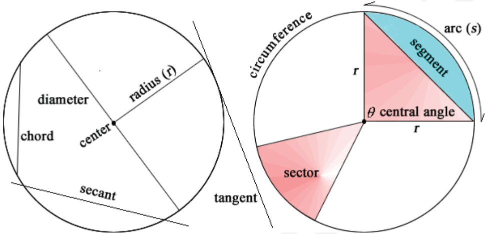
Perimeter: $P = a + c + b + d = a + c + \frac{h}{\sin(\alpha)} + \frac{h}{\sin(\gamma)}$

Area: $A = \frac{h}{2}(a+c)$

Height: $h = \frac{2A}{a+c}$

A trapezoid that has two pairs of parallel sides is a parallelogram, so rectangles, squares and rhombuses are also special types of trapezoid.

Geometry - Parts of a Circle



The **circumference** or perimeter of a circle is its boundary.

An **arc** is a part of the circumference of a circle.

The **center** of a circle is the point inside of the circle that is equidistant from all points on the boundary.

A **chord** is a line segment whose endpoints lie on the boundary of a circle.

A **secant** is a line that contains a chord of a circle.

A **diameter** of a circle is a line segment which passes through the center and whose endpoints lie on the boundary.

A **radius** of a circle is a line segment joining the center to any point on the boundary.

A **semicircle** is an arc that intersects with a diameter at its endpoints.

A **tangent** to a circle is a line that intersects the boundary of that circle at exactly one point.

A **sector** of a circle is a region that is bounded by two radii and an arc of the circle.

A **segment** of a circle is a region that is bounded by a chord and an arc of the circle.

A **central angle** of a circle is an angle that has its vertex at the center of the circle.

Radius (r), Diameter (d), Circumference (C), Area (A), π (pi) ≈ 3.14159

Diameter: $d = 2r$ **Circumference** (or Perimeter): $C = 2\pi r$ **Area**: $A = \pi r^2$

Radian Measure

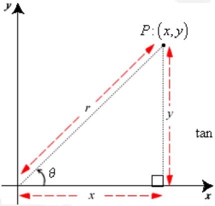
A circle in the coordinate plane with a radius of one unit and center at the origin is a **unit circle**. A **central angle** is an angle whose vertex is at the center of a circle. An **arc** is a portion of the circumference or boundary of a circle. If θ represents a central angle measured from the positive x-axis in a unit circle, then θ determines the length of the arc $\boldsymbol{s}$ on the circle that is bounded by the sides of the angle. The measure of angle θ is s **radians**. One radian is the measure of the central angle that bounds an arc on a unit circle that is equal in length to the radius of the circle. For a circle of radius r, the **radian measure** of an angle is the ratio of the length of the arc cut off by the angle's sides, s, to the length of the radius: $\theta = s/r$, where s and r are measured in the same units. The length of arc is $s = r\theta$.

Angle Conversions (Radians-Degrees)

2π rad $= 360°$ π rad $= 180°$ $\pi/2$ rad $= 90°$ $\pi/3$ rad $= 60°$ θ rad $= (\pi/180)\cdot\theta°$

$\pi/4$ rad $= 45°$ $\pi/6$ rad $= 30°$ $\pi/180$ rad $= 1°$ 1 rad $\approx 57.296°$ $\theta° = (180/\pi)$ rad

Definitions of the Six Basic Trigonometric Functions



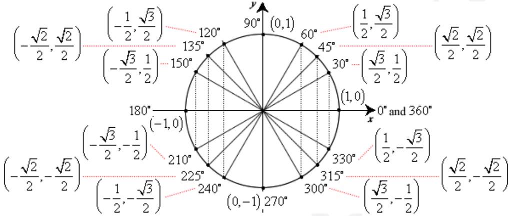
$$\sin(\theta) = \frac{y}{r} \qquad \operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y}$$

$$\cos(\theta) = \frac{x}{r} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x}$$

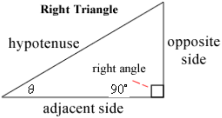
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y}$$

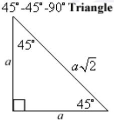
$$r = \sqrt{x^2 + y^2}$$

Trigonometry - Unit Circle, Even and Odd, Periodicity



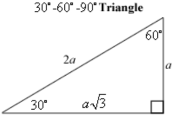
Right Triangles and Two Special Triangles





If $a = 1, a\sqrt{2} = \sqrt{2}$

If $a = \frac{1}{2}, a\sqrt{2} = \frac{\sqrt{2}}{2}$



If $a = 1, a\sqrt{3} = \sqrt{3}, 2a = 2$

If $a = \frac{1}{2}, a\sqrt{3} = \frac{\sqrt{3}}{2}, 2a = 1$

Even and Odd Functions

Cosine is an **even** function, so its values are symmetric with respect to the y-axis, and $\cos(-\theta) = \cos(\theta)$. Sine is an **odd** function, so its values are symmetric with respect to the origin, and $\sin(-\theta) = -\sin(\theta)$. Since tangent is a combination of sine and cosine, it is also an odd function, and $\tan(-\theta) = {}^{\sin(-\theta)}/_{\cos(-\theta)} = -\tan(\theta)$.

Periodicity

The basic sine and cosine functions take on all values between −1 and 1 inclusive, as their arguments cycle through all values from 0 to 2π (0° to 360°). Just as the angle-coordinate pair values on the unit circle repeat after each complete cycle, so do those of the sine and cosine functions. Sine and cosine both have periods of 2π and tangent has a period of π. If k is any whole number, then:

$\sin(\theta \pm 2k\pi) = \sin(\theta)$, $\cos(\theta \pm 2k\pi) = \cos(\theta)$, $\tan(\theta \pm k\pi) = \tan(\theta)$

Cofunction Identities

1 $\sin(\theta) = \cos(90° - \theta)$ **2** $\cos(\theta) = \sin(90° - \theta)$ **3** $\sec(\theta) = \text{cosec}(90° - \theta)$
4 $\text{cosec}(\theta) = \sec(90° - \theta)$ **5** $\tan(\theta) = \cot(90° - \theta)$ **6** $\cot(\theta) = \tan(90° - \theta)$

Pythagorean Identities

1 $\sin^2(\theta) + \cos^2(\theta) = 1$ **2** $\sec^2(\theta) - \tan^2(\theta) = 1$ **3** $\text{cosec}^2(\theta) - \cot^2(\theta) = 1$

Addition Formulas

1 $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
2 $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
3 $\tan(\alpha \pm \beta) = [\tan(\alpha) \pm \tan(\beta)]/[1 \mp \tan(\alpha)\tan(\beta)]$

Double-Angle Identities

1 $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
2 $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
3 $\tan(2\theta) = 2\tan(\theta)/[1 - \tan^2(\theta)] = 2/[\cot(\theta) - \tan(\theta)]$

Half-Angle Identities

The sign (+ or −) is determined by the quadrant the angle θ is in.

1 $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$ **2** $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$

Half-Angle Identities (continued)

3 $\tan\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}=\frac{\sin(\theta)}{1+\cos(\theta)}=\frac{1-\cos(\theta)}{\sin(\theta)}$

Power-Reduction Formulas

1 $\sin^2(\theta)=\frac{1}{2}\left[1-\cos(2\theta)\right]$

2 $\cos^2(\theta)=\frac{1}{2}\left[1+\cos(2\theta)\right]$

3 $\tan^2(\theta)=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Product-to-Sum and Product-to-Difference Identities

1 $\sin(\alpha)\sin(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$

2 $\cos(\alpha)\cos(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

3 $\sin(\alpha)\cos(\beta)=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

4 $\cos(\alpha)\sin(\beta)=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$

5 $\tan(\alpha)\tan(\beta)=\frac{\tan(\alpha)+\tan(\beta)}{\cot(\alpha)+\cot(\beta)}=\frac{\tan(\alpha)-\tan(\beta)}{\cot(\alpha)-\cot(\beta)}$

General guidelines in solving math problems

1 Read the problem completely first, to gain a general familiarity with it.

2 Read it through again, taking note of: **a** what kind of information is given (quantities specified etc.), **b** what information is being asked for, and **c** what process or processes to use.

3 Write down all of the important steps, keeping everything neat and legible. If there are any side calculations necessary to complete major steps, keep those together, neatly arranged and legible, in a separate section of your worksheet. Work only as quickly as you can and still keep your handwriting legible – you may have to retrace your steps.

4 If you are solving a computational problem involving equations and/or inequalities, neatly copy all of those onto your worksheet first. It can help you to keep track of every symbol being used.

5 Read each statement of your solution, rethinking any assumptions you may have made and any conclusions that you may have reached. Check all of your computations carefully – it isn't unheard of, that at some point in the rush of scribbling, that $12 \div 3 = 3$.

6 Ask yourself whether your answer makes sense, given the statement of the problem, then work your problem in reverse, substituting your solution(s) into the original problem.

7 To solidify your understanding of that type of problem, work as many similar ones as possible, taking note of the similarities in process, special constants that may appear and even the form of the solution.